The Role of Beauty in Research

A few thoughts for Ron Fagin celebrations

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Ron Fagin: what we have seen today

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- it affected both theory and practice of computing.

I want to talk about another aspect of Ron's work: it's elegance/beauty.

- Why is this important?
- We strive to find beautiful solutions.
- But are they necessary to solve problems?
- Do they please only their creators/inventors/discoverers?

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- Do we really care?
  - Who are we? Researchers? Users?
  - And what is beauty in the first place?

What is beauty?

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This is beautiful:

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Sometimes it seems to be universal
This certainly isn't:

What is beauty?

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But sometimes it's rather individual:

What is beauty?

It is universal or individual?

- Plato's view: universal
- Hume's view: individual
- I have to be a Scottish patriot and subscribe to Hume's view!

Getting closer to science

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The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colours or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics.

G H Hardy (1941)

Why do we need it?

- Beautiful definition: crystallize the concept
- Beautiful idea:
  - make it manageable
  - make it attractive to people
- Beautiful proof:
  - opens up new directions
  - sometimes simplicity leads to practical benefits

First example: relational databases

- Early days: messy models — network, hierarchical
  - hard to represent data, hard to query without knowing how it is organized
- Codd 1969: relational model. A beautiful concept:
  - separates logical and physical structure
  - logical structure: a fundamental mathematic concept – relations
  - querying language: first-order logic (FO)

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- No one has done more for the employment of logicians
  - Finite model theory: the backbone of database theory – thank you, Ron!

A more recent example: data exchange

- The subject took off in 2003.
- Reason: a very elegant paper FKMP03 that provided us with a good definition of data exchange.
- Good because:
  - it is a very realistic model of the problem,
  - it can be given by a very clean and concise mathematical definition (*"no ugly mathematics"*)
  - it connects nicely with well-known and studied concepts (tgds, egds, chase).
  - It's something you look it, understand it right away, like it, and start working with.

Data exchange cont'd

The rest is history. After hundreds of papers and several books, is there a single member of the community who has not seen this slide?

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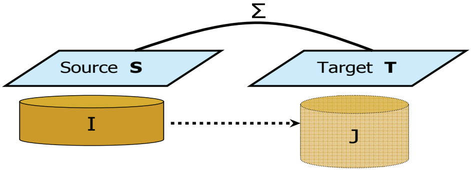
Schema Mappings & Data Exchange



- Schema Mapping **M** = (**S**, **T**, Σ)
  - Source schema **S**, Target schema **T**
  - High-level, declarative assertions Σ that specify the relationship between **S** and **T**.
- Data Exchange via the schema mapping **M** = (**S**, **T**, Σ)
  Transform a given source instance I to a target instance J, so that (I, J) satisfy the specifications Σ of **M**.

Beautiful definitions

- Always strive to find one
- They may pay off
- Sometimes sooner, sometimes later, sometimes never
    - this is research, one never knows
- But ugly (just-before-deadline) definitions won't!

- You won't find a single example of an ugly definition in Ron's work.

What's next? A beautiful idea/concept

- A nice definition is not enough, we need to know how to use it
- For example, relational model comes equipped with
  - a new notion: querying becomes declarative
  - Key language for querying: first-order logic (FO)
  - Nice math comes to the rescue again with procedural languages that are implemented by DBMSs
  - Not out of nowhere (relation algebra as an algebraization of FO)

Personal favorite from Ron's work — Top-k

The definition is there:

- objects are ranked according to m criteria
- individual grades are aggregates: if x_i is the grade of x according to the ith criterion, we want to compute the aggregate $F(x_1, \ldots, x_m)$
- Task: find top k objects.

Ron's contribution: a new class of algorithms (FA, TA)

- Clean and beautiful solutions
- and practical too.
- What's truly amazing, they fit on one slide!

Fagin's Algorithm (FA)

1. Do sorted access in parallel to each of the m lists according to each criterion.
    - Stop when there are at least k objects, each of which have been seen in all the lists.
2. For each object x that has been seen:
    - Retrieve all of its fields $x_1, \ldots, x_m$ by random access.
    - Compute $F(x_1, \ldots, x_m)$.
3. Return the top k answers.

A very optimal Threshold Algorithm (TA)

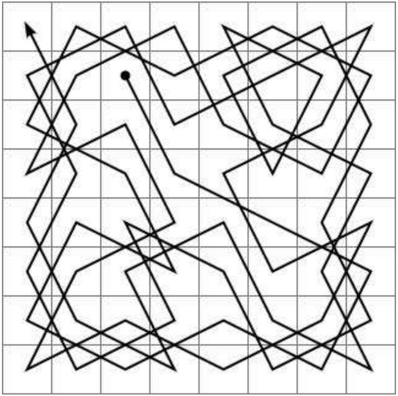
1. Do sorted access in parallel to each of the m lists according to each criterion. As each object x is seen under sorted access:
   - Retrieve all of its fields $x_1, \ldots, x_m$ by random access.
   - Compute $F(x) = F(x_1, \ldots, x_m)$.
   - If this is one of the top k answers so far, remember it.
2. For each ordered list according to criterion i, let $\hat{x}_i$ be the grade of the last object seen under sorted access.
3. Define the threshold value t to be $F(\hat{x}_1, \ldots, \hat{x}_m)$.
4. When k objects have been seen whose grade is at least t, then stop.
5. Return the top k answers.

Beautiful proofs

- *"Beauty is the first test: there is no permanent place in this world for ugly mathematics."* (G H Hardy)
- We want a nice and clean argument
- Often we are not satisfied with hitting it with all the hammers we've got until it works
  - although the deadline-driven "culture" makes us do precisely that too often...
- The notion of what is beautiful here may be more controversial and dependent on one's background.
- A couple of examples now.

Chess problem: a cute short (and overused) proof

It is well known that a knight can cover the entire chessboard without visiting the same square twice.



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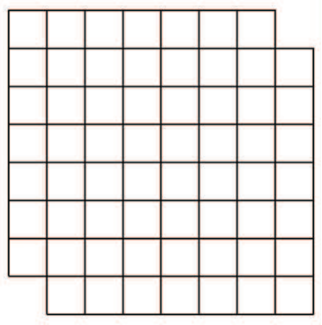
But what if we remove two squares in opposite corners?



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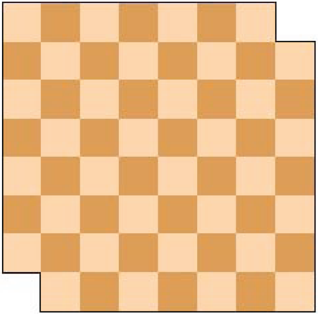
Solution: remember colors of squares



Opposite corners have the same color.

A knight move changes color – so no chance!

More controversial: there are infinitely many primes

1. Euler's product formula:

$$\prod_{p \text{ prime}} \frac{1}{1 - 1/p^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

2. If there were finitely many primes, the product would be a rational number, and hence π would be algebraic
3. But π is transcendental, so there are infinitely many primes.

A beautiful proof can really change things

- 0-1 law for first-order logic: most of the things you want to know are really boring (at least at infinity)
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- Pick a database "at random".
- Check if it satisfies a property $\mathcal{P}$.
- What's the probability of that?
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- If $\mathcal{P}$ is expressed in FO (or relational calculus/algebra), it is 0 or 1: 0-1 law.
- Pick a graph at random:
  - throw n vertices
  - for each pair of vertices toss a coin to see if they are connected
  - compute the probability for each n and see how it behaves as $n \longrightarrow \infty$

The 0-1 law for FO story

- First proved by 4 Russians (Glebskii, Kogan, Liogonki, and Talanov) in 1969
- The proof was very proletarian
  - emphasis on heavy tools, weight rather than technique
- English translations appeared in the early 1970s, but were very hard to follow.
- Ron Fagin could not follow them, and came up with a beautiful proof.
- Sounds like a recipe. Take:
  - a bit of probability
  - a bit of combinatorics
  - a bit of logic
- mix them quickly and get the result.

Fagin's proof

- The probability bit. Look at the statement $EA_{n,m}$

  for any disjoint sets X and Y with n and m nodes in a graph, there is a node v connected to everything in X and to nothing in Y

  - with probability 1 all such statements are true
- The combinatorial bit (that goes infinite). There is exactly one countable graph **G** that satisfies all the $EA_{n,m}$s.
- The logic bit. A first-order sentence is true with probability 1 iff it is true in **G** (traditional proof via compactness).
- Since in a concrete structure (like **G**) every sentence is either true or false, the result follows.
- Magic!

0-1 laws: what happened later

- Fagin's proof gave a methodology for proving 0-1 laws.
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- We now have them for many logics (e.g., fixed-point logics)
- We also have them for complex probability distributions
- For instance, with n vertices we can put edges with probabilities $\frac{1}{n^\alpha}$, for $0 < \alpha < 1$.
- An amazing result by Spencer-Shelah: FO has the 0-1 law iff α is irrational.
- There are lots of papers and books on the subject.

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Do all beautiful results have impact? Of course not.

But most definitions/ideas/results that have a big impact are indeed nice and beautiful: all you've seen today confirms it.

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- We must enjoy what we do!
- If we don't, we stop being researchers.
- It's hard to enjoy ugly things – and it is easy to enjoy beautiful ones.

Two side remarks

- Teaching — some of the favorite things to teach come from Ron's work:
  - 0/1 law;
  - top-k
  - Connectivity not in ∃MSO: another magical proof that only requires color chalk and an eraser to teach.
- Because beauty can be enjoyed indefinitely!
- Personal experience: finite model theory book.

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In Ron we have an example of someone with a perfect eye for scientific beauty — thank you very much for letting others appreciate what you've done!